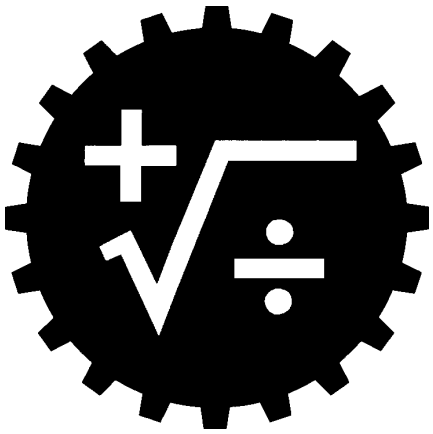
# MISSOURI'S

## Framework for Curriculum Development In

# Mathematics

## K-12



Missouri Department of Elementary and Secondary Education
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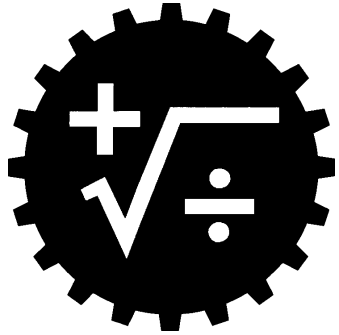
# I. Problem Solving



## K-12 Content Overview

Problem solving should be the central focus of the mathematics curriculum. As such, it is a primary goal of all mathematics instruction and an integral part of all mathematics activity. Problem solving is not a distinct topic but a process that should permeate the entire program and provide the context in which concepts and skills can be learned. Posing and solving problems should be the major focus of all students' mathematical activity so that through working with interesting, engaging, and intellectually stimulating situations, they come to understand mathematics and use it effectively.

Experiences should be such that students use discovery-oriented, inquiry-based and problem-centered approaches to investigate and understand mathematics. Students should be able to recognize, formulate, clarify and engage in solving problems arising from mathematical situations, everyday experiences, applications to other disciplines, and real-world applications.

# I. Problem Solving

| What All Students Should Know | What All Students Should Be Able To Do | Sample Learning Activities |
|---|---|---|
| *By the end of grade 4, all students should know*<br><br>1. A variety of problem-solving strategies (such as making a list, drawing a picture, looking for a pattern, acting out the problem).<br><br>2. Computational strategies with whole numbers (addition, subtraction, multiplication and division).<br><br>3. When to use concrete objects, calculators, computers, charts, graphs, etc., to organize and solve problems.<br><br>4. Mathematical problem-solving strategies can apply to all disciplines and real-world problems. | NOTE: Each item in this column is designed to address several elements of "what all students should be able to do."<br><br>*By the end of grade 4, all students should be able to*<br><br>a. work individually and with others to use problem- solving approaches to investigate and understand mathematical content (NCTM Standard 1; MO 1.6, 3.5, 3.6, 4.6)<br><br>b. use problem solving strategies to construct meaning from mathematical tasks (NCTM Standard 1; MO 1.6, 3.7)<br><br>c. recognize and define theoretical and actual problems encountered in everyday life, mathematical situations, and various disciplines (NCTM Standard 1; MO 3.1, 3.4)<br><br>d. develop and apply strategies to predict, prevent, and solve a wide variety of problems (NCTM Standard 1; MO 3.2, 3.3)<br><br>e. verify, interpret, and evaluate whether a solution addresses the original problem (NCTM Standard 1; MO 2.2, 3.6, 3.7, 3.8)<br><br>f. select and apply appropriate mathematical tools and technology to solve problems (NCTM Standard 1; MO 2.7) | NOTE: Each activity is designed to address several items from "what all students should know" and "what all students should be able to do." The activities may also relate to strands other than problem-solving.<br><br>• Given objects and/or pictures that have a variety of attributes such as shapes, colors, and sizes, devise a rule for sorting. Sort the objects and/or pictures using that rule. In spoken or written form explain the rule used (for example, "I put the blue ones together" or "all the big ones belong here.").<br><br>• Sort or classify a set of three-dimensional objects (such as blocks, soup cans, cereal boxes) by common attributes (including sides, ability to roll, and size).<br><br>• Given a menu from a fast-food restaurant, list five ways that you and a friend could eat for $5.00. Compute the cost of each of the five ways. |

# I. Problem Solving

| What All Students Should Know | What All Students Should Be Able To Do | Sample Learning Activities |
|---|---|---|
| | | • Boxes of the same height are stacked on top of each other in a storeroom. A first set of boxes is 8 inches high, a second set of boxes is 12 inches high, and a third set of boxes is 16 inches high. Design a strategy to determine the height when the tops of all the boxes are even. The height of the storeroom is 10 feet. Is there more than one time when the tops of the boxes would be the same height? Justify your answer.<br><br>• Given an advertisement from a local toy store, write a story problem that could be solved using both addition and subtraction. Solve the problem.<br><br>• Put six small counters under one hand, and without looking, move four of them into view. Can you figure out how many are still under your hand? Share your solution with a classmate. |



MATHEMATICS K–4

# I. Problem Solving

| What All Students Should Know | What All Students Should Be Able To Do | Sample Learning Activities |
|---|---|---|
| *By the end of grade 8, all students should know*<br><br>1. A variety of problem-solving strategies (such as organizing data, drawing a picture, looking for a pattern, writing an expression using a variable).<br><br>2. Computational strategies with whole numbers, decimals, fractions, and integers.<br><br>3. Models, calculators, computers, charts, and graphs may be used to organize and solve problems.<br><br>4. Mathematical problem-solving strategies can apply to all disciplines and real-world problems. | NOTE: Each item in this column is designed to address several elements of "what all students should be able to do."<br><br>*By the end of grade 8, all students should be able to*<br><br>a. use problem-solving approaches to explore and construct meaning from mathematical content (NCTM Standard 1; MO 1.6, 3.4, 3.5, 3.7)<br><br>b. pose authentic problems within and outside the field of mathematics (NCTM Standard 1; MO 2.1, 3.1, 3.4, 3.6)<br><br>c. develop and apply strategies to solve problems, with emphasis on multistep and non-routine problems (NCTM Standard 1; MO 1.8, 3.2, 3.3)<br><br>d. analyze, evaluate, and verify results with respect to the original problem (NCTM Standard 1; MO 2.2, 3.7, 3.8)<br><br>e. transfer strategies to similar problems (NCTM Standard 1; MO 3.2, 3.3, 3.6) | NOTE: Each activity is designed to address several items from"'what all students should know" and "what all students should be able to do." The activities may also relate to strands other than problem-solving.<br><br>• Identify a fast-food restaurant that offers a "Value Meal" combo. Investigate the restaurant's prices to determine whether the "Value Meal" or a $1.00 off coupon (purchasing the same items) is a better deal. Determine your city's sales tax rate, how the tax is added to the dollar total, and decide if this additional cost affects your solution.<br><br>• Consider the days of the month as "even" days and "odd" days (for example, Jan. 1 is "odd," Jan. 2 is "even"). If you received $1.00 on "odd" days and 50 cents on "even" days, on which day would you have accumulated $50? Explain your process. List the patterns you see in finding the $50 solution. Describe a general rule to determine the amount of money accumulated given a date.<br><br>• Given a 30 cm by 30 cm grid sheet, find the largest volume possible if you were to make an open-ended box (five sides with the top open). Make a chart showing the results of your investigations. Once the largest volume is found, be prepared to justify the results. Change the original grid sheet size, such as, 20 by 20, 20 by 30, and look for patterns in the results. Determine a general rule to describe the results. |

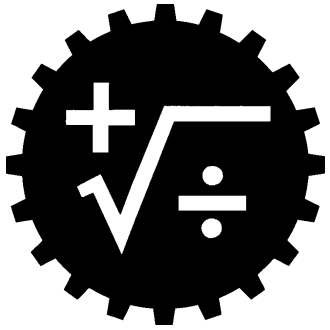
# I. Problem Solving

| What All Students Should Know | What All Students Should Be Able To Do | Sample Learning Activities |
|---|---|---|
| | | • Create a scale model of your mathematics classroom. Determine the total area of the room that can be painted. Explain how you would determine the amount of wall space that is to be painted. Investigate the amount of paint necessary to appropriately cover an interior wall. Calculate the number of cans of paint necessary to cover the walls in your mathematics classroom. Estimate the amount of paint necessary to paint another classroom in your school. Can you use your original calculations to estimate the amount of paint needed to paint the interior of your entire school? Explain your reasoning.<br><br>• A national magazine, *"Middle Schoolers R Us,"* has reported that an 8th grader's potential walking speed is determined by his/her height. Develop a method to test this claim. Conduct the appropriate experiments, then prepare a report of your results. The report should include charts and graphs in addition to the calculations justifying the conclusions. Assuming that speed is a function of height and this is a linear relationship, consider the following: (1) A new student (e.g., Slim Pickens, height 6'5") enrolls in your school. Determine his speed potential. (2) Explain whether a speed of 0 is realistic. (3) Investigate the real-world realistic values for height (domain) and the real-world realistic values for speed (range). Then justify your conclusions.<br><br>• Plan a home-cooked meal and a similar meal from a fast-food restaurant. Determine which meal is healthier based on nutritional value (percent of fat, percent of salt, etc.). Determine the cost of each meal. Justify whether you should eat out or eat the home-cooked meal. |

# I. Problem Solving

| What All Students Should Know | What All Students Should Be Able To Do | Sample Learning Activities |
|---|---|---|
| *By the end of grade 12, all students should know* | NOTE: Each item in this column is designed to address several elements of "what all students should be able to do."<br><br>*By the end of grade 12, all students should be able to* | NOTE: Each activity is designed to address several items from"'what all students should know" and "what all students should be able to do." The activities may also relate to strands other than problem-solving. |
| 1. Problem-solving strategies such as organizing data, drawing a picture, looking for a pattern, modeling, researching, and algebraic strategies. | a. use problem-solving strategies to investigate and understand mathematical content (NCTM Standard 1; MO 1.6, 3.5 | • Develop a way to enhance the marketability of a concert, including the number, cost, and types of tickets sold. |
| 2. Computational strategies for the set of real numbers. | b. recognize and formulate problems from situations within and outside mathematics (NCTM Standard 1; MO 3.1, 3.5) | • Develop a brochure and a cost analysis to be used by a sales force for selling tour packages with options. |
| 3. Models, calculators, computers, charts, graphs, etc., may be used as problem-solving tools. | c. organize, develop and apply integrated mathematical problem-solving strategies to solve problems within and outside mathematics (NCTM Standard 1; MO 3.2, 3.3) | • Create a game for a Mathematics Carnival. Investigate and justify the mathematical principles involved in the game.<br><br>• Design a procedure to examine the cost-effectiveness of games for a Mathematics Carnival. |
| 4. Mathematical problem-solving strategies can apply to all disciplines and real-world problems. | d. apply the process of mathematical modeling to real-world problem situations (NCTM Standard 1; MO 2.1, 3.6) | • Compare and contrast the mathematics and methods used for solving similar problems from different eras. |
| | e. analyze, evaluate, and reflect upon the process(es) used in solving problems (NCTM Standard 1; MO 2.2, 3.4, 3.6, 3.7, 3.8) | • Given a limited budget, calculate the cost of all materials needed to build a garage, an addition to a room, or a renovation. Create an itemized list of the amount and cost of the materials needed for the project. Specify and justify the dimensions of the garage or the added room or the renovation.<br><br>• Estimate the amount of concrete needed for a sidewalk 10 feet long. Describe how you determined your estimate. |

# II. Communication



## K-12 Content Overview

Mathematics can be thought of as a language that must be meaningful if students are to communicate mathematically and apply mathematics productively. Communication plays an important role in helping students construct links between their informal, intuitive notions and the abstract language and symbolism of mathematics; it also plays a key role in helping students make critical connections among physical, pictorial, graphic, symbolic, verbal and mental representations of mathematical ideas. When students see that one representation, such as an equation, can describe many situations, they begin to understand the power of mathematics. When they realize that some ways of representing a problem are more helpful than others, they begin to understand the flexibility and usefulness of mathematics.

Classroom experiences should allow students to model situations and discuss mathematical ideas using oral, written, concrete, pictorial, graphical, and algebraic methods, and a variety of technologies, such as computers, calculators, video, CD-ROM, and videodisks to represent and communicate their mathematical ideas.

# II. Communication

| What All Students Should Know | What All Students Should Be Able To Do | Sample Learning Activities |
|---|---|---|
| *By the end of grade 4, all students should know*<br><br>1. The language of mathematics may be used in reading, writing, listening and speaking.<br><br>2. Mathematical ideas may be represented by visual models.<br><br>3. Mathematical symbols represent real-world situations.<br><br>4. Information may be organized in a variety of ways. | NOTE: Each item in this column is designed to address several elements of "what all students should be able to do."<br><br>*By the end of grade 4, all students should be able to*<br><br>a. relate physical materials, pictures, and diagrams to mathematical ideas (NCTM Standard 2; MO 2.1)<br><br>b. organize information into useful forms, such as verbal, symbolic, or graphic (NCTM Standard 2; MO 1.8)<br><br>c. reflect on and clarify thinking about mathematical ideas and situations (NCTM Standard 2; MO 2.2)<br><br>d. communicate the relationship between everyday language, mathematical language and symbols (NCTM Standard 2; MO 2.3)<br><br>e. demonstrate the ability to select and apply appropriate strategies, such as representing, discussing, reading, writing, listening, and using technology in mathematics (NCTM Standard 2; MO 2.2) | NOTE: Each activity is designed to address several items from 'what all students should know" and "what all students should be able to do." The activities may also relate to strands other than communication.<br><br>• Use manipulatives (such as, color tiles, or Unifix cubes) to demonstrate the meaning of addition, subtraction, multiplication, and division.<br><br>• Write, tell, or illustrate a story using mathematical language to describe real-world situations.<br><br>• Investigate the number of rectangles that can be made using a set number of color tiles. Communicate your results.<br><br>• Design a display for, temperature, rainfall amounts, and/or growth of a plant.<br><br>• Read a story or listen to a song and illustrate or discuss the mathematical concepts discovered there.<br><br>• Save the seeds from a piece of watermelon. Count the number of seeds from your watermelon and add your information to a class graph. Read the graph to see whose watermelon had the most seeds and whose had the least seeds. |

# II. Communication

## What All Students Should Know

*By the end of grade 8, all students should know*

1. The language of mathematics may be used through reading, writing, listening, and speaking.
2. How to represent mathematical ideas with visual models.
3. Mathematical symbols may be used to represent a variety of situations.
4. That information may be organized in a variety of ways.

## What All Students Should Be Able To Do

NOTE: Each item in this column is designed to address several elements of "what all students should be able to do."

*By the end of grade 8, all students should be able to*

a. model situations using oral, written, concrete, pictorial, graphical, technological, and algebraic methods (NCTM Standard 2; MO 1.8, 2.1)

b. reflect on and model mathematical ideas and mathematical situations common to the classroom and the workplace (NCTM Standard 2; MO 2.6, 4.8)

c. reflect on and clarify their own thinking about mathematical ideas and situations (NCTM Standard 2; MO 2.2)

d. develop common understanding of mathematical ideas, including the role of definitions (NCTM Standard 2; MO 2.2, 2.3)

e. draw mathematical ideas and conclusions from reading, listening, and viewing (NCTM Standard 2; MO 3.5, 4.1)

f. discuss mathematical ideas, make conjectures, and present convincing rationales (NCTM Standard 2; MO 2.4)

g. connect mathematical notation and its role in the development and structure of mathematical ideas (NCTM Standard 2; MO 1.6, 1.9, 2.4)

## Sample Learning Activities

NOTE: Each activity is designed to address several items from 'what all students should know" and "what all students should be able to do." The activities may also relate to strands other than communication.

- Make a diagram to represent the following situation: There are 30 students in an algebra class. Eleven of these students are in band and 15 play basketball. Five students are in both band and basketball. Write a narrative to explain your diagram.

- Design a spinner in two colors, blue and green. Make the spinner in such a way that it is twice as likely to land on blue as on green. Explain your rationale for the way you designed your spinner.

- Given a dart board with the point values of only 4 and 7, and an unlimited number of darts, find the largest impossible score. Describe in writing why the given solution must be the largest impossible score.

- Use the information below to construct a mathematical problem and write a narrative of how to solve your problem.

| Ice cream | Cost per serving | Flavor rating |
|---|---|---|
| Cherry Berry | $1.25 | 7 |
| Chocolate Rave | .89 | 4 |
| White Sparkle | .95 | 3 |
| Candy Cane | 1.09 | 3 |
| Fruit 'n' Fun | .85 | 7 |

# II. Communication

| What All Students Should Know | What All Students Should Be Able To Do | Sample Learning Activities |
|---|---|---|
| *By the end of grade 12, all students should know*<br><br>1. The language of mathematics may be used through reading, writing, listening, and speaking.<br><br>2. Mathematical ideas may be represented with visual models.<br><br>3. Mathematical symbols may be used to represent a variety of situations.<br><br>4. Information may be organized in a variety of ways. | NOTE: Each item in this column is designed to address several elements of "what all students should be able to do."<br><br>*By the end of grade 12, all students should be able to*<br><br>a. reflect upon and clarify thinking about mathematical ideas and relationships (NCTM Standard 2; MO 1.6, 2.2)<br><br>b. interpret generalizations discovered through investigations to formulate, revise, and adjust mathematical definitions (NCTM Standard 2; MO 1.2, 1.7, 2.2)<br><br>c. visualize mathematical ideas by reading about, listening to, or viewing concrete models (NCTM Standard 2; MO 1.9, 2.4)<br><br>d. plan and create effective verbal and non-verbal forms of communicating mathematics for a variety of purposes and audiences (NCTM Standard 2; MO 2.1)<br><br>e. present mathematical ideas and logical justifications, both written and oral (NCTM Standard 2; MO 2.1, 3.5, 4.1)<br><br>f. ask clarifying and extending questions about the mathematics read about, heard about, or viewed through models (NCTM Standard 2; MO 2.3) | NOTE: Each activity is designed to address several items from"'what all students should know" and "what all students should be able to do." The activities may also relate to strands other than communication.<br><br>• Read a piece of literature such as *Gulliver's Travels* or *Through the Looking Glass* looking for relationships in mathematics. Discuss how mathematics is used in the literature. Write an essay or short story describing situations, visual images and objects in terms from mathematics using both numerical relationships and geometric relationships.<br><br>• Design a three-dimensional scale model to illustrate a given structure such as a shopping mall, an amusement park, or a sports arena.<br><br>• Design an acceptable popcorn container to hold the greatest amount of popcorn and use the least possible amount of materials.<br><br>• Contact a business such as a financial institution, construction company, local industry, or chamber of commerce to collect information so you can prepare charts and graphs representing information collected for a presentation or to convince a customer to buy your product.<br><br>• Using number sets such as natural, whole integer, rational, irrational, and real numbers, design some diagram, model or video to describe the relationships of the sets. |

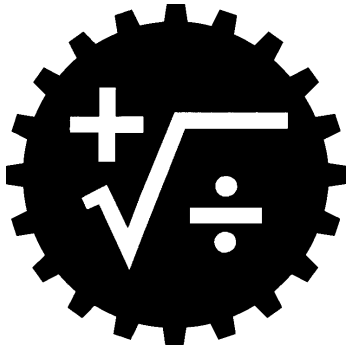
# II. Communication

| What All Students Should Know | What All Students Should Be Able To Do | Sample Learning Activities |
|---|---|---|
| | g. recognize the economy, power, and elegance of mathematics notation and its role in the development of mathematical ideas (NCTM Standard 2; MO 1.6, 1.9, 2.4)<br><br>h. read, write, and talk about mathematical ideas as they relate to real-life applications and multiple workplace situations (NCTM Standard 2; MO 1.10, 2.6, 3.2, 4.8) | |

## II. Communication

| What All Students Should Know | What All Students Should Be Able To Do | Sample Learning Activities |
| --- | --- | --- |
| | | |

# III. Reasoning



## K-12 Content Overview

Every student should have the opportunity to develop higher order thinking skills. The key is to unlock the world of mathematics through a student's natural inclination to strive for purpose and meaning. Reasoning is fundamental to the knowing and doing of mathematics. Demonstrating the logical validity of conjectures is the essence of the creative act of doing mathematics. To give more students access to mathematics as a powerful way of making sense of the world, it is essential that an emphasis on reasoning pervade all mathematical activity. Also important is the goal that all students become confident, self-reliant mathematical thinkers. Students need to develop the capability to confront a mathematical problem, persevere in its solution, and evaluate and justify their results.

Students should learn to appreciate the pervasive use and power of reasoning as part of mathematics, develop the habit of monitoring and validating their own thinking, and use their mathematical reasoning skills in other disciplines in their lives.

# III. Reasoning

| What All Students Should Know | What All Students Should Be Able To Do | Sample Learning Activities |
|---|---|---|
| *By the end of grade 4, all students should know*<br><br>1. Objects/numbers may be used in more than one way to determine or construct relationships between and among them.<br><br>2. Results must be verified.<br><br>3. Data may be organized in a variety of forms to look for patterns.<br><br>4. Geometric and number properties. | NOTE: Each item in this column is designed to address several elements of "what all students should be able to do."<br><br>*By the end of grade 4, all students should be able to*<br><br>a. draw logical conclusions about mathematics (NCTM Standard 3; MO 3.5)<br><br>b. use models, known facts, properties, and relationships to explain their thinking (NCTM Standard 3; MO 4.1)<br><br>c. justify answers and solution processes in an organized and convincing way (NCTM Standard 3; MO 1.8, 3.4, 3.7, 4.1)<br><br>d. use patterns and relationships to analyze mathematical situations (NCTM Standard 3; MO 1.6) | NOTE: Each activity is designed to address several items from 'what all students should know" and "what all students should be able to do." The activities may also relate to strands other than reasoning.<br><br>• Given the following statements, find the secret number:<br>(a) The number is odd.<br>(b) The number is less than 40 and greater than 30.<br>(c) The number is not 33.<br>(d) The sum of the digits is between 5 and 10.<br><br>• Prepare a set of three clues similar to the ones presented in activity #1 that will describe a secret number.<br><br>• What could be the next number in the sequence, "2, 4,..."? Explain your answer. Can you justify any other numbers? Explain all possibilities you find.<br><br>• Determine how much food a given pet would eat in a year and estimate the total cost. Explain how you reached your answer.<br><br>• Play card games such as "War" or "Double War." Tell how you decided who won the game. |

# III. Reasoning

| What All Students Should Know | What All Students Should Be Able To Do | Sample Learning Activities |
|---|---|---|
| | | • Play strategy games such as tic-tac-toe, NIM games, checkers, chess or other games requiring the use of or development of winning strategies. Explain your winning strategies for the game you selected.<br><br>• Play "Guess My Rule" with a set of blocks or objects. One person thinks of an attribute (color, shape or size). The others determine the chosen attribute by placing the blocks within a designated area. Objects are placed in or out of the area until enough information is acquired to guess the attribute. |



MATHMATICS K–4

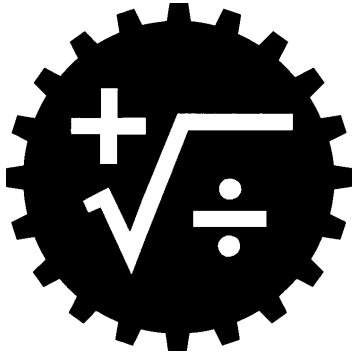
# III. Reasoning

| What All Students Should Know | What All Students Should Be Able To Do | Sample Learning Activities |
|---|---|---|
| *By the end of grade 8, all students should know*<br><br>1. Information may be organized in a variety of forms to look for patterns and relationships.<br><br>2. Results must be justified.<br><br>3. Geometric and number properties. | NOTE: Each item in this column is designed to address several elements of "what all students should be able to do."<br><br>*By the end of grade 8, all students should be able to*<br><br>a. make and interpret mathematical conjectures and rationales for a conclusion (NCTM Standard 3; MO 1.5)<br><br>b. justify their own thinking (NCTM Standard 3; MO 4.1, 4.4)<br><br>c. use reasoning processes in regard to spatial reasoning and reasoning with proportions and graphs (NCTM Standard 3; MO 1.6, 1.8, 3.5)<br><br>d. recognize and apply deductive and inductive reasoning (NCTM Standard 3; MO 3.5, 4.1)<br><br>e. use patterns and relationships to generalize an algebraic representation (NCTM Standard 3; MO 1.6) | NOTE: Each activity is designed to address several items from 'what all students should know" and "what all students should be able to do." The activities may also relate to strands other than reasoning.<br><br>• Collect data on the number of windows on one floor or part of your school and estimate how many windows are in the building. Justify your reasoning.<br><br>• Look at patterns of numbers and determine possible next terms in the pattern. Predict the general formula that fits the pattern.<br><br>• Make a scale drawing of a roller coaster and sketch a graph to represent the speed of a roller coaster versus its position on the track. Explain the reasoning used to construct your graph. Interpret the graph.<br><br>• Use or generate data to predict some future result. Continue the experiment to test conjectures. Analyze the results by comparing conjectures to results and describe how (and if) a revision is necessary to improve future predictions. |

# III. Reasoning

| What All Students Should Know | What All Students Should Be Able To Do | Sample Learning Activities |
|---|---|---|
| *By the end of grade 12, all students should know*<br>1. Information may be organized in a variety of forms to look for patterns and relationships.<br>2. Results must be justified.<br>3. Geometric and number properties.<br>4. The process of inductive reasoning.<br>5. The process of deductive reasoning. | NOTE: Each item in this column is designed to address several elements of "what all students should be able to do."<br>*By the end of grade 12, all students should be able to*<br>a. make and test conjectures (NCTM Standard 3; MO 1.7)<br>b. defend the validity of their conclusions using mathematical strategies (NCTM Standard 3; MO 3.4, 3.7, 3.8, 4.1)<br>c. follow the mathematical reasoning of others and determine validity (NCTM Standard 3; MO 1.5, 2.3)<br>d. apply inductive and deductive reasoning (NCTM Standard 3; MO 3.5) | NOTE: Each activity is designed to address several items from"'what all students should know" and "what all students should be able to do." The activities may also relate to strands other than reasoning.<br>• Prepare an organized and convincing report analyzing the effect of the national debt on the country, state, community and family.<br>• Evaluate and discuss the validity of mathematical reasoning by the authors of news articles; bring in articles for presentations before the class in which the author's mathematical validity, or lack thereof, is analyzed.<br>• Demonstrate how categorizing, drawing schematics, or other organizing systems help to facilitate the reasoning process. Examples include the properties of numbers, categorization of geometric shapes, paths and graph theory. |

# IV. Connections



## K-12 Content Overview

Although it is often necessary to teach specific concepts and procedures, mathematics must be approached as a whole. Concepts, procedures and intellectual processes are interrelated. In a very real sense, "the whole is greater than the sum of its parts." This strand includes a deliberate attempt, through specific learning activities, to connect ideas and procedures both among different mathematical topics and with other content areas.

Instructional strategies should allow students to view mathematics as an integrated whole; link conceptual and procedural knowledge; understand and use various representations of concepts and connect them to one another; explore problems and describe and confirm results using various representations; use models, calculators, and other technologies to connect different representations of mathematical concepts; and use one mathematical idea to extend understanding of another, related mathematical idea.

Students should be able to connect mathematics to the real world by recognizing the connection between mathematics and other disciplines and applying mathematical thinking and problem-solving in those areas. They should also recognize the role of mathematics in their daily lives, in careers, and in society and apply mathematics in those contexts.

# IV. Connections

## What All Students Should Know

*By the end of grade 4, all students should know*

1. Problems may be looked at in more than one way.

2. Mathematics is used in other subject areas.

3. Mathematics is used in the real world.

## What All Students Should Be Able To Do

NOTE: Each item in this column is designed to address several elements of "what all students should be able to do."

*By the end of grade 4, all students should be able to*

a. link concepts to student-generated procedures (NCTM Standard 4; MO 1.6, 1.10, 2.2)

b. relate various representations of concepts or procedures to one another using a variety of methods, forms, and technologies (NCTM Standard 4; MO 1.6, 2.7)

c. recognize relationships among different topics in mathematics (NCTM Standard 4; MO 1.6, 1.10)

d. use mathematics in other curriculum areas and in daily living (NCTM Standard 4; MO 1.10, 4.7)

## Sample Learning Activities

NOTE: Each activity is designed to address several items from"'what all students should know" and "what all students should be able to do." The activities may also relate to strands other than connections.

- Read literature such as *Anno's Counting House, Ten Black Dots, The Button Box, 17 Kings 42 Elephants, How Big is a Foot, The Door Bell Rang*. (Many activities involving mathematics will arise during the reading of such books.)

- Given a floor plan of your school, find the shortest walking path from your classroom to the cafeteria. Then find the shortest path to the cafeteria that goes by a rest room.

- Your school's parent group has donated funds to purchase new playground equipment. Design a model of the equipment and the playground.

- Create a time-line for the history of the school.

- Collect or record data gathered in other disciplines.

- Explore numeric and geometric patterns.

# IV. Connections

| What All Students Should Know | What All Students Should Be Able To Do | Sample Learning Activities |
|---|---|---|
| | | • After attending a career/hobby day, discuss how mathematics is used in the careers and hobbies you investigated.<br><br>• Identify real-world situations in which mathematics is used. |

# IV. Connections

| What All Students Should Know | What All Students Should Be Able To Do | Sample Learning Activities |
|---|---|---|
| *By the end of grade 8, all students should know*<br><br>1. Problems may be looked at in more than one way.<br><br>2. Mathematics is used in other subject areas.<br><br>3. Mathematics is used in the real world. | NOTE: Each item in this column is designed to address several elements of "what all students should be able to do."<br><br>*By the end of grade 8, all students should be able to*<br><br>a. investigate problems utilizing graphical, numerical, physical, algebraic, and mathematical models or representations to understand and describe conclusions (NCTM Standard 4; MO 1.2, 1.6, 2.1, 3.7)<br><br>b. apply mathematical reasoning and modeling to solve problems from other disciplines, such as art, music, psychology, science, and business (NCTM Standard 4; MO 1.2, 1.3, 3.2, 3.5)<br><br>c. use technology such as scientific and graphing calculators, computers and models to demonstrate understanding of mathematical ideas (NCTM Standard 4; MO 1.4, 2.7)<br><br>d. explore and investigate the importance of mathematics in their lives, future careers, and our ever-changing global society (NCTM Standard 4; MO 4.2, 4.3, 4.7, 4.8) | NOTE: Each activity is designed to address several items from "what all students should know" and "what all students should be able to do." The activities may also relate to strands other than connections.<br><br>• Interpret a graphical representation or model, such as a pie chart, indicating music preference of students at a local middle school, and portray the information using a different representation or numerical model, physical, or algebraic.<br><br>• Investigate the mathematics (such as patterns and geometry) involved in a real-life situation, for example, why airlines use "hub" cities in scheduling their routes, or why pictures/frames are manufactured to certain proportions.<br><br>• Use mathematical models and ratios to investigate the relationship between naturally occurring situations such as wing span vs. body length for several bird species. Given the data generated, predict the wing span for a new species of bird or predict arm span for a person with a specific height.<br><br>• Investigate, then utilize, the Conservation Department's methods of estimating the number of fish of a certain species in an area lake. Justify your results and discuss how you could verify your predictions. |

# IV. Connections

| What All Students Should Know | What All Students Should Be Able To Do | Sample Learning Activities |
|---|---|---|
| | | • Utilize current technology to investigate how changing the scale on a graph, or using a different representation such as bar graphs, pie graphs, or box graphs, may alter the appearance and impact of the data.<br><br>• Investigate data from the previous 10-20 years involving local or state citizens, such as quantity of fresh water consumed annually, number of schools/classrooms needed, or number of housing units needed; then predict the amount that will be necessary 10 years in the future, considering population trends. |



MATHEMATICS 5–8

# IV. Connections

## What All Students Should Know

*By the end of grade 12, all students should know*

1 Problems may be looked at in more than one way.

2. Mathematics is used in other subject areas.

3. Mathematics is used in the real world.

## What All Students Should Be Able To Do

NOTE: Each item in this column is designed to address several elements of "what all students should be able to do."

*By the end of grade 12, all students should be able to*

a. recognize and/or derive equivalent representations for a concept (NCTM Standard 4; MO 1.6)

b. analyze and relate procedures in multiple representations (NCTM Standard 4; MO 1.5, 3.6)

c. relate and describe the connections within topics of mathematics and other disciplines (NCTM Standard 4; MO 1.6, 1.8, 1.10)

d. investigate and determine the importance of mathematics in their lives, future careers, and our ever-changing global society (NCTM Standard 4; MO 4.1, 4.2, 4.3, 4.4, 4.5, 4.6, 4.7, 4.8)

e. evaluate the logic and aesthetics of mathematics as they relate to the universe (NCTM Standard 4; MO 1.10, 2.4)

## Sample Learning Activities

NOTE: Each activity is designed to address several items from"'what all students should know" and "what all students should be able to do." The activities may also relate to strands other than connections.

- Find the Olympic gold medal times for the men's 100 meter dash and display the data in a table and scatter plot. Use a graphics calculator to calculate and display the curve of best fit. Predict the gold medal time for the year 2020.

- Investigate the volume of an open-ended box that may be constructed from poster board measuring 20 by 30 centimeters. Discuss the method(s) used to find the dimensions allowing for the maximum volume. Justify your solution.

- Use land surveys to calculate areas using two different methods. Explain, contrast, and compare the two methods.

- Investigate recursion by comparing a head of broccoli to its flowerets and the flowerets to its subsets. Write about or discuss the patterns observed. Do further research on recent findings in the area of fractals, chaos and recursion.

- Investigate the volume of your local phone book. Estimate the total volume of all phone books to be recycled in your school and community. Research the amount of landfill rescued as a result of this recycling project. Compare and contrast disposal methods to project the cost benefits to your community and the environment.

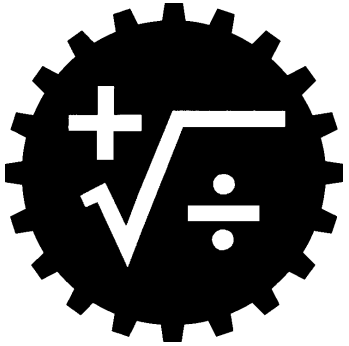
# IV. Connections

| What All Students Should Know | What All Students Should Be Able To Do | Sample Learning Activities |
|---|---|---|
| | | • Play Monopoly by the rules, but pay for transactions by writing checks (Four to six students per game.)<br>1. Select a banker.<br>2. Distribute money by the rules of the game.<br>3. Players must fill out deposit slips to deposit the money in the bank.<br>4. Players keep track of transactions on a bank record sheet. The banker also keeps track of all the transactions for all the players in the game.<br>5. Students may keep some small bills to pay fines. All other transactions must be completed by check.<br>6. Students reconcile their bank statements with their records.<br>7. The winner is determined by the rules of the game. The assessment is based on the students' banking records and whether the checks were written correctly.<br>NOTE: Contact a local bank to receive banking materials (check books with checks, deposit slips, and check registers).<br>• List five occupations that never use mathematics and/or computers. List the mathematics actually used in an occupation that seems mathematics-free such as a gift wrapper in a department store. Write a paragraph describing the career you might want to pursue and the use of mathematics and/or computers in that occupation. |

# IV. Connections

| What All Students Should Know | What All Students Should Be Able To Do | Sample Learning Activities |
|---|---|---|
| | | • Investigate, analyze, and predict workplace safety using data from OSHA.<br><br>• Investigate the impact of a 5¢ national sales tax considering your family, local community, state, and nation.<br><br>• Use the stock market section of the newspaper to select the stock of a company to follow. Keep track of the stock for several weeks. Make charts and graphs to display the information. Analyze the data to determine the trends for increases and decreases. Justify your analysis. |

# V. Number Sense
# (Show-Me Standards, Math 1)



## K-12 Content Overview

Number sense is a quality that successful users of mathematics possess. It is often identified as an intuitive feel for numbers and their variety of interpretations as well as a common sense approach to using numbers. When someone chooses to use fractions in one situation and decimals in another because the respective operations are easier to perform or the results are easier to understand, that process is evidence of good number sense. When students continue to work on a problem until they recognize that their answers are reasonable in the problem's context, they are using good number sense. Intelligent use of the decimal number system and powers of 10 and the ability to use approximations and estimation when appropriate are other indicators of a well-developed number sense.

Computation should support meaningful experiences in geometry, probability, measurement, and other areas of mathematics. Estimation should be used to solve problems for which exact answers are inappropriate and for checking computation results. As students begin to understand the meaning of operations and develop a concrete basis for validating symbolic processes and situations, they should design their own algorithms and discuss, compare and evaluate them. Students should analyze the ways the various algorithms work and how they relate to the meaning of the operation and to the numbers involved.

Number sense is not inherent in a person's ability to perform numerical computations. A "sense-building mode" is best established when students are provided with opportunities to explore number relationships, are encouraged to challenge and question, and are allowed to experiment to discover strategies and techniques of their own that ease the path to the solution of mathematical problems.

Students will only develop strong number sense to the extent that their teachers use pedagogy that encourages the understanding of mathematics as opposed to the memorization of rules and mechanical application of algorithms. Every child has the capability to succeed as a user of mathematics but the degree of success is directly related to the strength of their number sense. The way to ensure that all students acquire a good sense of number is to have them consistently engage in activities that require them to think about numbers and number relationships and to make the connection with quantitative information encountered in their daily lives.

# V. Number Sense

| What All Students Should Know | What All Students Should Be Able To Do | Sample Learning Activities |
|---|---|---|
| *By the end of grade 4, all students should know*<br>1. Counting and grouping strategies.<br>2. Mental computation and estimation strategies.<br>3. Place value.<br>4. Basic computation facts of addition, subtraction, multiplication, and division with whole numbers.<br>5. Addition and subtraction of fractions with like denominators.<br>6. U.S. customary and metric units of measure.<br>7. The appropriate use of calculators. | NOTE: Each item in this column is designed to address several elements of "what all students should be able to do."<br>*By the end of grade 4, all students should be able to*<br>a. model, explore, develop, and explain number operations for whole numbers (NCTM Standard 7; MO 1.6, 2.1, 3.3)<br>b. use technology to explore numbers (NCTM Standard 6; MO 1.4, 1.6, 2.7)<br>c. use physical models and real-world experiences to construct number meanings (NCTM Standard 5; MO 1.10, 2.3, 4.1)<br>d. demonstrate an understanding of our numeration system by relating counting, grouping, and place value concepts (NCTM Standard 6; MO 1.6, 3.6, 4.1)<br>e. utilize number sense to develop number meanings and explore number relationships (NCTM Standard 6; MO 1.6, 3.3) | NOTE: Each activity is designed to address several items from "what all students should know" and "what all students should be able to do." The activities may also relate to strands other than number sense.<br>• Use manipulatives such as color tiles, and unifix cubes to show the relationships among addition, subtraction, multiplication, and division.<br>• Use manipulatives to demonstrate an understanding of place value by modeling operations on numbers.<br>• Use manipulatives or models to demonstrate fractional parts and the fractional notations representing 1/2, 1/3, 2/4, etc. Show fractional parts that are equivalent. Explain how you know they are equivalent.<br>• Skip count using a calculator. Record the data. Examine the units place for patterns. Describe the patterns.<br>• Using a Frisbee or paper plate, estimate how far it can be thrown. Throw and give a better estimate. Now measure the actual distance. Repeat the experiment several times and average the data. |

# V. Number Sense

| What All Students Should Know | What All Students Should Be Able To Do | Sample Learning Activities |
|---|---|---|
| | f. use a variety of mental computation and estimation strategies to solve specific problems (NCTM Standard 5; MO 1.10, 3.3, 4.1)<br><br>g. demonstrate an understanding of the attributes of length, capacity, weight, area, volume, time, temperature, and angle (NCTM Standard 5; MO 1.6, 4.1)<br><br>h. make and use standard and nonstandard measurements in problems and everyday situations (NCTM Standard 5; MO 3.2, 3.3)<br><br>i. explore the concepts of fractions, mixed numbers, and decimals and be able to apply them to problem situations (NCTM Standard 12; MO 1.6, 3.2, 3.3, 4.1) | • Estimate the number of raisins in a 1/2 oz. box. Record estimates. Open the box and examine the raisins in view. (Do not empty the box.) Revise estimates if necessary. Count raisins. (A third estimate can be made when raisins are dumped from the box.)<br><br>• Create a chart to help the shipping clerks of Yummy Gum speed up their work. Yummy Gum ships gum by packs and single pieces. Each pack contains 10 pieces of gum. Create a chart or a fast way that the shipping clerks can package and send off the orders.<br><br>• With a partner, a bowl with 20 cubes and a die, guess the number of rolls it will take to empty the bowl. Roll the die and remove that number of cubes from the bowl. After rolling the die the number of times you guessed, record "yes" or "no" to indicate if the bowl was emptied. (from *About Teaching Mathematics*, 1992) |



MATHEMATICS K–4

# V. Number Sense

| What All Students Should Know | What All Students Should Be Able To Do | Sample Learning Activities |
|---|---|---|
| *By the end of grade 8, all students should know*<br><br>1. Addition, subtraction, multiplication, and division with rational numbers.<br><br>2.Numbers and their relationships can be represented in multiple forms.<br><br>3. The appropriate use of technology. | NOTE: Each item in this column is designed to address several elements of "what all students should be able to do."<br><br>*By the end of grade 8, all students should be able to*<br><br>a. extend understanding and ability to apply whole number operations to all rational numbers (such as integers, and fractions, decimals) (NCTM Standard 6; MO 3.2, 3.3, 4.1)<br><br>b. use multiple representations of equivalent forms of numbers such as integers, fractions, decimals, percent, exponents, and scientific notation in a variety of situations (NCTM Standard 5; MO 1.6, 1.10)<br><br>c. describe connections and relationships of numbers such as ratios, proportions, and percents in problem situations (NCTM Standard 5; MO 1.6, 2.1)<br><br>d. investigate number forms such as fractions, decimals, and percents, and demonstrate their use in today's society (NCTM Standard 5; MO 1.10, 2.2, 4.1) | NOTE: Each activity is designed to address several items from"'what all students should know" and "what all students should be able to do." The activities may also relate to strands other than number sense.<br><br>• Provided with retail lists, including prices and the sales tax rate, estimate the total cost of selected items. Explain your strategy.<br><br>• Explore the local forms of media to find examples of how fractions, decimals, and percents are used. Justify the appropriateness of the media's use of the numbers.<br><br>• Investigate the relationship between perimeter and area of closed figures and generalize the results.<br><br>• Using ratios, proportions, or a method of choice, predict a future situation, such as population of Missouri in the year 2010, the number of bald eagles nesting in Missouri in 2000, on the year someone will run the mile in 3 minutes flat.<br><br>• Aiming for a target interval, enter a number into a calculator. Use only the operation of multiplication to arrive at a number within the interval. Compare the impact of multiplying by a number less than one and a number greater than one.<br><br>• Select a rational number. Show three other representations of the same number. |

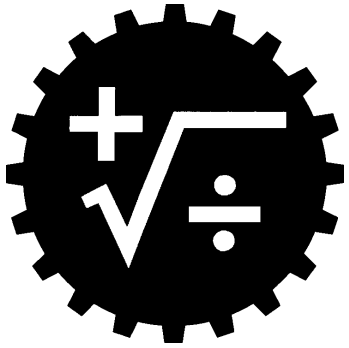
# V. Number Sense

| What All Students Should Know | What All Students Should Be Able To Do | Sample Learning Activities |
|---|---|---|
| | e. develop, analyze, and explain procedures for computation and techniques for estimation (NCTM Standard 7; MO 2.3, 3.3)<br><br>f. develop, analyze, and explain methods for solving proportions (NCTM Standard 7; MO 2.3, 3.3)<br><br>g. check and explain the reasonableness of solutions, strategies, and results (NCTM Standard 7; MO 1.7, 3.3, 3.7, 4.1)<br><br>h. select appropriate methods of computation, such as mental arithmetic, estimation, calculator, computer, and paper/pencil to reflect upon solutions, strategies and results (NCTM Standard 7; MO 3.3, 3.6, 3.7)<br><br>i. represent numerical relationships in one- and two-dimensional graphs (NCTM Standard 5; MO 2.1, 2.7) | • Play Monopoly by the rules, but pay for transactions by writing checks (four to six students per game).<br><br>1. Select a banker.<br><br>2. Distribute money by the rules of the game.<br><br>3. Players must fill out deposit slips to deposit the money in the bank.<br><br>4. Players keep track of transactions on a bank record sheet. The banker also keeps track of all the transactions for all the players in the game.<br><br>5. Students may keep some small bills to pay fines. All other transactions must be completed by check.<br><br>6. Students reconcile their bank statements with their records.<br><br>7. The winner is determined by the rules of the game. The assessment is based on the students' banking records and whether the checks were written correctly.<br><br>NOTE: Contact a local bank to receive banking materials (check books with checks, deposit slips, and check registers). |

# V. Number Sense

| What All Students Should Know | What All Students Should Be Able To Do | Sample Learning Activities |
|---|---|---|
| *By the end of grade 12, all students should know*<br><br>1. Addition, subtraction, multiplication, and division with real numbers.<br><br>2. Numbers and their relationships can be represented in multiple forms.<br><br>3. The appropriate use of technology.<br><br>4. Data can be organized in many forms. | NOTE: Each item in this column is designed to address several elements of "what all students should be able to do."<br><br>*By the end of grade 12, all students should be able to*<br><br>a. develop, analyze, and explain procedures used for representing and analyzing relationships in tables, verbal rules, equations, and graphs (NCTM Standards 5 and 6; MO 1.6, 1.8, 1.10, 2.6)<br><br>b. analyze the effects of parameter changes on the graphs of functions (NCTM Standards 5 and 6; MO 1.6, 3.6, 4.1, 4.7)<br><br>c. analyze and describe relationships and the resulting effects between changes in an independent variable and a dependent variable (NCTM Standards 5 and 6; MO 1.6, 3.3, 4.1) | NOTE: Each activity is designed to address several items from 'what all students should know" and "what all students should be able to do." The activities may also relate to strands other than number sense.<br><br>• Research and discuss the relationship of various currency systems and difficulties encountered in the exchange between systems.<br><br>• Examine the effects of changing parameters in compound interest formulas, growth patterns/half-life formulas, fractals and chaos equations. Describe how the changes affect a specific situation.<br><br>• Given a problem situation involving complex computations, determine a range within which a reasonable answer could be expected to fall. Explain why you selected the range.<br><br>• Discuss whether the method used to present mathematical data influences numerical sense of understanding (such as, different scales on the axes of a graph or changing window on a graphing calculator).<br><br>• Discuss the significance of a small numerical error compounded over a long distance or a long time (such as, a small angular error when traveling to the moon in a space ship). |

# VI. Geometric and Spatial Sense
Show-Me Standards, Math 2



## K-12 Content Overview

Geometry is the systematic study of spatial relationships. It is connected to every strand in a mathematics curriculum and to a vast multitude of situations in real life. Well-constructed diagrams allow us to apply knowledge of geometry and geometric reasoning and intuition to arithmetic and algebra problems. Whether designing an electronic circuit board, a building, a dress, an airport, a bookshelf or a newspaper page, a solid understanding of basic geometric principles is required.

Traditionally, geometry in schools has been taught as the prime example of a formal deductive system. While this view of the content is important, its domination has led to the exclusion of other, less readily formalized topics and applications. Geometry instruction should not be limited to formal deductive proof and simple measurement activities, but should include the study of geometric transformations, analytic geometry, topology, and the connection of geometry with algebra and other areas of mathematics.

By virtue of living in a three-dimensional world, having dealt with space for 5 years, children enter school with a remarkable amount of intuitive geometric knowledge. The geometry curriculum should take advantage of this intuition. In early elementary school, a rich, qualitative, hands-on study of geometric objects helps young children develop spatial sense and a strong intuitive grasp of geometric properties and relationships. Eventually they develop a comfortable vocabulary of appropriate geometric terminology. In the middle school years, students should begin to use their knowledge in a more analytical manner to solve problems, make conjectures, and look for patterns and generalizations. Gradually they develop the ability to make inferences and logical deductions based on geometric relationships. In high school, the study of geometry expands to include coordinate geometry, trigonometry, and both inductive and deductive reasoning.

# VVI. Geometric and Spatial Sense

| What All Students Should Know | What All Students Should Be Able To Do | Sample Learning Activities |
|---|---|---|
| | NOTE: Each item in this column is designed to address several elements of "what all students should be able to do." | NOTE: Each activity is designed to address several items from "what all students should know" and "what all students should be able to do." The activities may also relate to strands other than geometric and spatial sense. |
| *By the end of grade 4, all students should know* | *By the end of grade 4, all students should be able to* | |
| 1. Standard and nonstandard units of measure. | a. describe, model, draw, and classify shapes (NCTM Standard 9; MO 1.4, 1.6, 2.1) | • A dog is tied to a 5-meter rope at the middle of the 10-meter-long side of a garage. Make a sketch of the outer path on which the dog can walk. Move the tie post to the corner of the garage. Compare and contrast the outer path of the two situations. |
| 2. Descriptions of two- and three-dimensional figures. | b. investigate and predict the results of combining, subdividing, and changing shapes (NCTM Standard 9; MO 1.1, 1.6, 3.1) | • Classify shapes by matching concrete objects, such as tangrams, pattern blocks, or cutouts to their name. Create a design using one of each shape. |
| 3. Geometric shapes are found in the real world. | c. visualize, draw, and compare shapes (NCTM Standard 9; MO 1.8, 2.1, 3.2, 3.3) | • Given a model, use interlocking cubes to build a three-dimensional replica of the model. |
| 4. Objects can be located by relative position. | d. connect geometric ideas to number and measurement ideas (NCTM Standard 9; MO 1.6, 3.5, 4.1) | • Use a nonstandard unit of measure (such as beans) to find the area of a leaf. |
| 5. The process of measurement. | e. explore geometry in their world (NCTM Standard 9; MO 1.10, 2.4) | • Use a nonstandard unit of measure (such as beans, hands, feet, paper clips) to find a classmate's height; the height of your desk; the length and width of your desk; the area of a foot print, a hand or a leaf; the mass or weight of classroom objects (using a balance scale). |
| | f. investigate concepts of lines, angles, similarity, congruence, and symmetry (NCTM Standard 9; MO 1.6, 2.5) | |
| | g. investigate length, capacity, weight, mass, area, volume, time, and temperature (NCTM Standard 10; MO 1.6, 2.5) | |

| What All Students Should Know | What All Students Should Be Able To Do | Sample Learning Activities |
|---|---|---|
| | h. use standard and nonstandard units of measure (NCTM Standard 10; MO 1.10)<br><br>i. locate objects by relative position including top, bottom, left, right, over and under (NCTM Standard 9; MO 1.6) | • Explore the letters of the alphabet or geometric shapes (such as the shapes contained in a bucket of pattern blocks) for mirror symmetry. Sort the letters into categories that have one line of mirror symmetry, more than one line of mirror symmetry, and those letters that do not have mirror symmetry.<br><br>• Build a shape (two-dimensional) or structure (three-dimensional) from a description provided by another student. Build a shape or structure with a set of materials that cannot be seen by the other student. Another student with an identical set of materials tries to build the shape or structure with only verbal directions. (For younger children, teacher gives description.)<br><br>• Copy a partner's design onto another geoboard or dot paper.<br><br>• Make a shape (perhaps something that can fly) on your geoboard. Describe how shapes are alike and how they are different. Then sort and classify the shapes.<br><br>• Play "I'm Thinking of an Object." Give clues about the location of the object you are thinking about. The classmate who guesses what you are thinking about then takes a turn. |



MATHEMATICS K–4

# VI. Geometric and Spatial Sense

| What All Students Should Know | What All Students Should Be Able To Do | Sample Learning Activities |
|---|---|---|
| *By the end of grade 8, all students should know*<br>1. Structures of measurement systems.<br>2. Descriptions of two- and three-dimensional shapes and their relationships .<br>3. Geometric shapes are found in the real world. | NOTE: Each item in this column is designed to address several elements of "what all students should be able to do."<br>*By the end of grade 8, all students should be able to*<br>a. identify, describe, compare, classify, and represent geometric figures (NCTM Standard 12; MO 1.4, 1.6, 2.1)<br>b. explore transformations of geometric figures (NCTM Standard 12; MO 1.6)<br>c. investigate and apply geometric properties and relationships (NCTM Standard 12; MO 1.6, 2.4, 3.6)<br>d. use geometry to describe their world (NCTM Standard 12; MO 1.10, 2.4)<br>e. extend their understanding of the process and structure for measurement (NCTM Standard 13; MO 1.4, 2.6, 2.7)<br>f. select and discuss appropriate units and devices to estimate or make measurements, considering degree of accuracy (NCTM Standard 13; MO 2.6, 3.1, 3.7, 4.1) | NOTE: Each activity is designed to address several items from "what all students should know" and "what all students should be able to do." The activities also relate to strands other than geometric and spatial sense.<br>• Use straws to construct polyhedra and classify according to their attributes.<br>• Use modeling or simulation to compare the sum of the length of two sides of a triangle to the third side. Justify your results.<br>• Draw a single figure on graph paper. Use reflections, slides and turns to make a wall paper design.<br>• Collect examples of geometry used in architecture. Describe geometric characteristics such as the Golden Rectangle.<br>• Given 90 yards of fencing, design a fence to enclose a swimming pool to allow for the following: the largest possible area, the most visually pleasing shape, and the most creative shape. Justify how your shape fits each category. |

g. apply the concepts of perimeter, area, volume, angle measure, capacity, weight, and mass (NCTM Standard 13; MO 2.5, 3.8, 4.1)

h. investigate the concept of rate of change (NCTM Standard 13; MO 1.4, 1.6, 1.8)

i. develop formulas and procedures for determining measures to solve problems (NCTM Standard 13; MO 1.4, 1.6, 1.8, 3.7)

- Given a commercial cereal box, calculate the potential volume. Investigate how changing the dimensions of the box will affect the volume. Using the given amount of pasteboard, determine the largest possible volume. Discuss why dimensions of a current box are used instead of another dimension.

# VI. Geometric and Spatial Sense

| What All Students Should Know | What All Students Should Be Able To Do | Sample Learning Activities |
|---|---|---|
| *By the end of grade 12, all students should know*<br>1. Structures of geometric and measurement systems.<br>2. Properties and relationships of two- and three-dimensional shapes.<br>3. Geometric shapes can be used to describe the real world. | NOTE: Each item in this column is designed to address several elements of "what all students should be able to do."<br>*By the end of grade 12, all students should be able to*<br>a. interpret and draw three-dimensional objects (NCTM Standard 7; MO 1.5, 1.9, 2.7)<br>b. represent and solve problem situations with geometric models and apply properties of figures (NCTM Standard 7; MO 1.5, 2.7, 3.7)<br>c. classify figures in terms of congruence and similarity and apply these relationships (NCTM Standard 7; MO 1.1, 1.4, 1.6, 3.5)<br>d. deduce properties of, and relationships between, figures from given assumptions (NCTM Standard 7; MO 1.6, 1.8, 2.4, 3.5)<br>e. translate between synthetic and coordinate representations using a variety of methods and technologies (NCTM Standard 8; MO 1.4, 2.7)<br>f. deduce properties of figures using transformations and coordinates (NCTM Standard 8; MO 2.4, 3.5) | NOTE: Each activity is designed to address several items from "what all students should know" and "what all students should be able to do." The activities may also relate to strands other than geometric and spatial sense.<br>• Build models of the five regular polyhedra. Discuss the relationships between the number of faces, edges, and vertices. Through exploration, pose the question as to why there are only five regular polyhedra.<br>• Build three-dimensional objects following a model, then draw the front, right, top, and base views to demonstrate skills in spatial visualization with a partner. Each two-person team will then create a base view for another team to build.<br>• Determine the smallest amount of material needed to produce a can with a given volume Justify your results.<br>• Use paper-folding activities and/or computer technology to deduce properties and relationships between figures (such as exploring the relationships of opposite sides, opposite angles, diagonals of the quadrilateral family, relationships between angles, chords of a circle, etc.). Develop a simple deductive system using these relationships.<br>• Create a design to tile a floor or a wallpaper pattern using tessellations. |

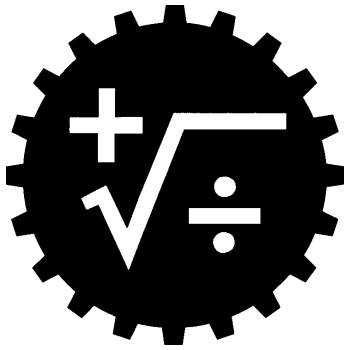
| What All Students Should Know | What All Students Should Be Able To Do | Sample Learning Activities |
|---|---|---|
| | g. identify congruent and similar figures using transformations (NCTM Standard 8; MO 1.5, 3.5, 3.6) | • Design a packing box that can be used to ship eight small spheres (such as decorative ornaments) using the least amount of material. Justify your results. |
| | h. analyze properties of transformations and relate translations to vectors (NCTM Standard 8; MO 1.6, 2.4, 3.6, 4.1) | • Determine the angle of depression between two markers with different elevations on a contour map. |
| | i. apply an understanding of perimeter, area, volume, angle measure, capacity, weight and mass (NCTM Standard 7; MO 2.5, 3.3, 4.1) | • Determine the area of an irregularly shaped region (such as plans from a registered land surveyor). |
| | j. model, describe, and analyze maximum and minimum points on a graph (NCTM Standard 13; MO 1.6, 2.1, 4.3, 4.7) | • Find the limit of the area of a circle by constructing an inscribed and/or a circumscribed convex polygon and determining its area. Repeat the process increasing the number of sides of the polygon. Explain why the limit is numerically sensible. |
| | k. model, describe, and analyze patterns of sequences through processes of geometric change, approximations, and limits (NCTM Standard 14; MO 1.6, 2.1, 4.3, 4.7) | • Draw a triangle on a sheet of graph paper. Draw images using a rotation, a reflection, and a translation. |
| | l. recognize and apply trigonometry to problem situations (NCTM Standard 9; MO 3.1, 3.6, 4.2, 4.8) | |



MATHEMATICS 9–12

# VII. Data Analysis, Probability and Statistics

Show-Me Standards, Math 3



## K-12 Content Overview

Probability is the study of random events. It is used in analyzing games of chance, genetics, weather prediction, and a myriad of other everyday events. Statistics is the mathematics we use to collect, organize and interpret numerical data. It is used to describe and to analyze sets of test scores, election results and shoppers' preferences for particular products. Probability and statistics are closely linked because statistical data are frequently analyzed to see whether some conclusions can legitimately be drawn about a particular phenomenon and also to make predictions about future events.

Understanding probability and statistics is essential in the modern world with the print and electronic media full of statistical information and its interpretation. The goal of mathematical instruction in this area should be to make students sensible, critical users of probability and statistics, able to apply their processes and principles to real-world problems. Students should be able to judge whether the statistics are meaningful and used appropriately in an argument in order to make a decision on that argument.

Elementary children are able to understand the probability of an event. Experiments leading to discussions about the difference between experimental and theoretical probability can be done by older elementary and middle grade students. Young children can start out as early as kindergarten with data collection, organization, and graphing and a focus on those skills, with obviously increasing sophistication, should last throughout their schooling. Whenever students look at data, there should be some question they are trying to answer or some position they are trying to support.

Probability and statistics hold the key for enabling our students to understand, process and interpret the vast amounts of quantitative data that exist all around them. To be able to judge the truth of a data-supported argument presented to them, to discern the believability of a persuasive advertisement that talks about the results of a survey of all of the users of a particular product, or to be knowledgeable consumers of the data-intensive government and electoral statistics that are ever-present, students need the skills that they can learn in a well-conceived probability and statistics curriculum strand.

| What All Students Should Know | What All Students Should Be Able To Do | Sample Learning Activities |
|---|---|---|
| *By the end of grade 4, all students should know*<br>1. Strategies to collect data.<br>2. Strategies to organize data.<br>3. Different ways of displaying data.<br>4. The appropriate display of data.<br>5. The appropriate use of technology. | NOTE: Each item in this column is designed to address several elements of "what all students should be able to do."<br>*By the end of grade 4, all students should be able to*<br>a. collect, organize, and describe data through the use of technologies and other resources (NCTM Standard 11; MO 1.1, 1.3, 1.4, 1.8)<br>b. construct, read, and interpret displays of data through verbal, nonverbal, symbolic, and graphic forms (NCTM Standard 11; MO 1.5, 3.3, 3.6, 4.1)<br>c. solve problems that require collecting and analyzing data (NCTM Standard 11; MO 2.3, 3.2, 3.3, 4.3)<br>d. explore concepts of chance (NCTM Standard 11; MO 1.6, 1.7, 4.3, 4.7) | NOTE: Each activity is designed to address several items from "what all students should know" and "what all students should be able to do." The activities also relate to strands other than data analysis/ probability and statistics.<br>• Collect data on a given topic and classify the data based on similarities.<br>• Use collected data to construct a variety of graphs or charts. Discuss which representation best displays the information.<br>• Make a prediction on an upcoming event based on data collected and analyzed from a past series of events.<br>• Given a spinner that is divided into four equal sections with two blue sections, one red section, and one green section, how many times would you expect to get green if you spin 20 times? Explain your answer. Now spin 20 times and record your results. Compare the results to your prediction.<br>• Make a spinner with four equal parts. Spin the spinner 20 times and record the number of times each area of the spinner comes up. Repeat the above with four unequal parts on a spinner. Record your data. |

# VII. Data Analysis, Probability and Statistics

| What All Students Should Know | What All Students Should Be Able To Do | Sample Learning Activities |
|---|---|---|
| | | • Make a graph of the colors of socks students in your class wear to school on a given day. Explain the graph to a partner.<br><br>• Create an attendance graph to collect data for the class lunch count. Interpret the graph to help your teacher complete the lunch count report.<br><br>• Collect and interpret information from an opinion graph (such as favorite color, cartoon, movie, place to eat, etc.).<br><br>• Each pair of students needs 5 pennies and 5 dimes, a recording sheet for each and a piece of 8 1/2 x 11 paper (target). With your partner, toss all 10 coins onto the target paper. Then record in the appropriate column on the recording sheet the number of dimes and pennies that landed on the target.<br><br>• When the sheet is filled, add up the amount of money that hit the target for each player to find out who is the winner. |

| What All Students Should Know | What All Students Should Be Able To Do | Sample Learning Activities |
|---|---|---|
| *By the end of grade 8, all students should know*<br><br>1. Standard measures of central tendency.<br><br>2. Methods to analyze data.<br><br>3. Methods of representing analyzed data.<br><br>4. Similarities and differences in theoretical and experimental probabilities.<br><br>5. The appropriate use of technology. | NOTE: Each item in this column is designed to address several elements of "what all students should be able to do."<br><br>*By the end of grade 8, all students should be able to*<br><br>a. develop, analyze, and explain methods utilized to collect, organize, and describe data (NCTM Standard 10; MO 1.1, 1.4, 1.8, 2.1)<br><br>b. make, read, and interpret multiple representations including tables, charts and graphs of data (NCTM Standard 10; MO 1.5, 1.8, 3.3)<br><br>c. formulate, predict, and defend positions taken that are based on data collected (NCTM Standard 10; MO 1.2, 1.4, 2.1, 3.7)<br><br>d. analyze information and arguments that are based on data collected (NCTM Standard 10; MO 1.7, 3.4, 3.6)<br><br>e. investigate the power of making decisions based on statistical methods and the applications of probability in the real world (NCTM Standard 10; MO 1.3, 3.2, 4.3, 4.7)<br><br>f. use computers, graphing calculators, and/or other forms of technology to enhance understanding of numbers, data, and the resulting analysis (NCTM Standard 10; MO 1.4, 2.7) | NOTE: Each activity is designed to address several items from "what all students should know" and "what all students should be able to do." The activities also relate to strands other than data analysis/probability and statistics.<br><br>• Organize student-generated data from investigations of real-world situations (such as height vs. shoe size, predicted vs. actual colors in a bag, or height vs. how fast a person runs) into table, charts, and graphs. Describe the relationships observed from the data.<br><br>• Locate historical data (such as results from athletic events, annual precipitation for the county, or population) and prepare a graphic representation. Analyze the results to make predictions for the future.<br><br>• Prepare a set of questions and responses for a survey that will be given to fellow students to determine opinions on a current event. Distribute and collect the surveys, then analyze and display the results.<br><br>• Given a media representation of data, write a narrative for the information.<br><br>• Given data from a scientific experiment, organize the data into a symbolic or graphic representation. |

# VII. Data Analysis, Probability and Statistics

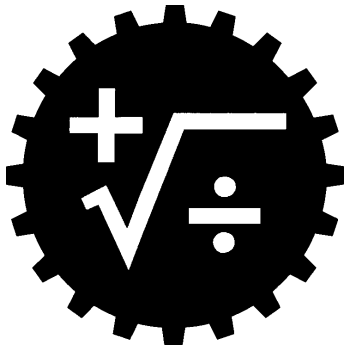
| What All Students Should Know | What All Students Should Be Able To Do | Sample Learning Activities |
|---|---|---|
| | g. develop and execute experiments or simulations to predict and determine probable outcomes (NCTM Standard 10; MO 1.2, 1.3, 3.1, 3.6) | • Randomly generate pairs of numbers with a sum ranging from 2 through 12 via number cubes, graphic calculators, or computers. Vary the sample size. Organize the results into graphical form. Describe the results. |
| | h. investigate sample spaces to predict probable outcomes and how these predictions affect the decision-making process (NCTM Standard 10; MO 1.3, 1.7, 1.10, 3.6) | • Given the 1-week daily attendance to an amusement park, swimming pool or water park and the corresponding weather reports, predict the daily attendance for a different week, given the 7-day weather forecast. Compare predicted results to actual results. |
| | i. investigate appropriate applications for experimental and theoretical probabilities (NCTM Standard 10; MO 1.7, 3.8, 4.7) | |

| What All Students Should Know | What All Students Should Be Able To Do | Sample Learning Activities |
|---|---|---|
| *By the end of grade 12, all students should know*<br><br>1. Statistical measures of central tendency, randomness, variability, and correlation.<br><br>2. Appropriate use of theoretical and experimental probabilities.<br><br>3. The process required to design and conduct a survey or experiment.<br><br>4. The process required to analyze and present data.<br><br>5. The appropriate use of technology. | NOTE: Each item in this column is designed to address several elements of "what all students should be able to do."<br><br>*By the end of grade 12, all students should be able to*<br><br>a. interpret and summarize data from charts, tables, and graphs that appear in real-world situations (NCTM Standard 10; MO 1.1, 1.8)<br><br>b. apply curve-fitting to make defendable predictions (NCTM Standard 10; MO 1.4, 2.7, 3.2)<br><br>c. apply the appropriate statistical measures including central tendency, variability, and correlation to a situation (NCTM Standard 10; MO 1.2, 1.5, 3.2)<br><br>d. investigate the effects of data transformations on variability and measures of central tendency (NCTM Standard 10; MO 1.1, 1.4, 2.7) | NOTE: Each activity is designed to address several items from "what all students should know" and "what all students should be able to do." The activities may also relate to strands other than data analysis/ probability and statistics.<br><br>• Investigate a municipal or county service (such as, energy, water, or trash collection) and determine usage over the past 20 years. Predict the quantity or usage 10 years from now. Analyze and describe how conservation and recycling can affect your prediction.<br><br>• Collect data from experimentation, a survey, or use published data to determine measures of center, spread, and extrema and use statistical measures to compare and make predictions about the future.<br><br>• Determine and implement methods for conducting an unbiased survey (such as a phone survey for political candidate).<br><br>• Organize data in a chart, stem/leaf, or box plot, and use technology to determine the line or curve of best fit.<br><br>• Design models to simulate relative frequency and probability of an event occurring.<br><br>• Determine the relative frequency of an event and relate it to theoretical probability. |

# VII. Data Analysis, Probability and Statistics

| What All Students Should Know | What All Students Should Be Able To Do | Sample Learning Activities |
|---|---|---|
| | e. investigate the concept of a random variable (NCTM Standard 10; MO 1.4, 2.7, 3.2)<br><br>f. design and interpret simulations to estimate probabilities (NCTM Standard 11; MO 1.3, 3.3, 3.6)<br><br>g. apply theoretical probability to real-world problems (NCTM Standard 11; MO 1.7, 3.8)<br><br>h. apply experimental probability to real-world problems (NCTM Standard 11; MO 1.7, 3.8)<br><br>i. collect, plot, and interpret data, including that from a discrete probability distribution (NCTM Standard 11; MO 1.2, 1.6, 3.6)<br><br>j. develop, interpret, and apply the normal curve in problem solving (NCTM Standard 11; MO 1.1, 3.2, 3.4)<br><br>k. determine and interpret maximum and minimum values within a data set, on a graph, or in a problem situation (NCTM Standard 13; MO 1.3, 2.1, 3.6)<br><br>l. analyze an infinite series as it relates to a limiting value (NCTM Standard 13; MO 1.6, 1.8, 3.2) | • Investigate a career that uses data analysis and make a presentation based on your finding.<br><br>• Determine the odds of an event occurring.<br><br>• Apply data collection, representation of data, and interpretation of data to solve real-world problems.<br><br>• Design and conduct a field or laboratory study by gathering and interpreting data, formulating a hypothesis, and expressing the results in a variety of presentations. |

# VIII. Patterns and Relationships
Show-Me Standards, Math 4



## K-12 Content Overview

Many mathematicians regard mathematics as the "science of patterns." Investigating the patterns that they find in numbers, shapes, and expressions is perhaps the most successful strategy students can use to make true mathematical discoveries. In so doing, they will be solving problems, dealing with important mathematical concepts and relationships, making and verifying generalizations, and constructing an initial understanding of a fundamental mathematical idea–the function.

All of the content strands have close interconnections, but this one is very closely tied to all of the others since an understanding of patterns can be either content or process. When the patterns themselves and their rules for generation are objects of study, they represent the content being learned. There is a very special relationship, though, between patterns and algebra. Algebra provides the language in which we communicate the patterns in mathematics.

Children become aware of patterns very early in their lives. Repetitive daily routines and periodic phenomena are all around them. In the early school grades, children need to build on those early experiences by constructing, recognizing, and extending patterns in a whole variety of contexts. Numbers and shapes certainly offer many opportunities, but so do music, language and physical activity. Children should construct their own patterns with manipulatives such as pattern blocks, attribute blocks and multilink cubes. The calculator is also a very useful tool for making the connection between counting patterns and rules of arithmetic operations.

In the higher elementary grades, input/output activities that require recognition of relationships between one set of numbers and a second set provide an early introduction to functions. In the middle grades, students begin to work with patterns that can be used to solve problems within the domain of mathematics as well as from the real world. Graphing software is extremely valuable at this level to help students visualize the relationships they discover.

At the secondary level, students are able to bring more of the tools of algebra to the problem of analyzing and representing patterns and relationships. The use of functions in modeling real-life observations also plays a central role in the high school mathematics experience. Graphing calculators and computers should be available to all students for use in line- and curve-fitting as approaches to the explanation of a set of experimental data .

# VIII. Patterns and Relationships

## What All Students Should Know

*By the end of grade 4, all students should know*

1. Mathematical ideas may be represented with visual models.
2. Mathematical symbols can be used to represent real-world situations.
3. Patterns and relationships can be represented in a variety of ways.
4. Information can be organized to look for a pattern or relationship.
5. Patterns can be geometric and/or numeric.

## What All Students Should Be Able To Do

NOTE: Each item in this column is designed to address several elements of "what all students should be able to do."

*By the end of grade 4, all students should be able to*

a. create, recognize, describe, and extend a wide variety of patterns (NCTM Standard 13; MO 1.6, 1.8, 2.1, 3.3)

b. represent and describe mathematical relationships (NCTM Standard 13; MO 1.6, 1.8, 2.2, 3.3)

c. investigate the use of variables and open sentences in expressing relationships (NCTM Standard 13; MO 1.6, 1.8, 3.3)

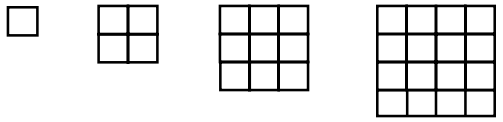
## Sample Learning Activities

NOTE: Each activity is designed to address several items from "what all students should know" and "what all students should be able to do." The activities may also relate to strands other than patterns and relationships.

- Given a picture of a design in a quilt, describe all the patterns you see. Explain what could be added to the design to generate yet another pattern.
- Use square tiles to make the first four rectangles in the design below. Count the number of squares in each rectangle and the number of units around each rectangle. Write a rule you might use to find the number of squares and the number of units around for the fifth rectangle.

a.

b.



- Given a hundreds chart with a few numbers darkened on the first two rows, complete a pattern over the entire chart. Describe or classify the numbers that have been darkened over the entire chart. Extension: Given a blank number hundreds chart and counting disks, make an original pattern. Change the pattern by moving a designated number of counting disks. Describe both patterns and their differences.

| What All Students Should Know | What All Students Should Be Able To Do | Sample Learning Activities |
|---|---|---|
| | | • Using the constant function on a calculator, construct an input/output table of numbers. Describe the relationships. Graph the results.<br><br>• Repeat a rhythm pattern begun by the teacher (such as clap-clap-clap-stamp-stamp-stamp-clap-clap-clap- or snap-clap-snap-clap-snap). Now create your own rhythm pattern and ask a classmate to repeat it.<br><br>• Practice initiating or extending geometric patterns or codes (such as circle-square-circle-square or AB-AAB-AAAB).<br><br>• Line up with six or seven of your classmates according to a pattern (such as boy/girl; short/tall). Ask other students to name the pattern(s) they see. |



MATHEMATICS K–4

# VIII. Patterns and Relationships

| What All Students Should Know | What All Students Should Be Able To Do | Sample Learning Activities |
|---|---|---|
| *By the end of grade 8, all students should know*<br><br>1. Mathematical ideas may be represented with visual models.<br><br>2. Mathematical symbols can be used to represent real-world situations.<br><br>3. Patterns and relationships can be represented in a variety of ways.<br><br>4. Information can be organized to look for a pattern or relationship.<br><br>5. Patterns can be geometric and/or numeric. | NOTE: Each item in this column is designed to address several elements of "what all students should be able to do."<br><br>*By the end of grade 8, all students should be able to*<br><br>a. examine, predict, design, extend, and describe patterns and relationships (NCTM Standard 8; MO 1.6, 1.8, 2.1, 3.3)<br><br>b. design and compare patterns and relationships using rules, charts, and graphs that may be constructed using technology (NCTM Standard 8; MO 1.6, 1.8, 2.7, 3.3)<br><br>c. examine patterns, relations, and functions to determine how a change in the independent variable can produce a change in a dependent variable (NCTM Standard 8; MO 1.6, 1.8, 3.3, 3.6)<br><br>d. apply patterns, relations, and functions to solving real-world problems (NCTM Standard 8; MO 1.6, 1.8, 3.3, 3.6)<br><br>e. solve equations and inequalities (NCTM Standard 9; MO 1.6, 1.8, 2.2, 3.3) | NOTE: Each activity is designed to address several items from "what all students should know" and "what all students should be able to do." The activities also relate to strands other than patterns and relationships.<br><br>• Predict the next three terms in the following sequence:<br><br>-5, -8, -11, -14, ... Explain how you reached your answer. Use your model to determine the 100th term.<br><br>• Make a diagram to show the maximum number of pieces of pizza made with three cuts across the center, four cuts across the center, and five cuts across the center. Find an explicit formula relating the number of cuts to the maximum number of pieces of pizza.<br><br>• Collect three flowers that grow in the Fibonacci pattern. Make a sketch of the petal pattern and discuss how it is within the Fibonacci set.<br><br>• List the first five rows in Pascal's triangle. Devise a pattern to symbolically write the next row in terms of the first five rows.<br><br>• Apply patterns of chaos to real-world situations (for example, shore lines). |

| What All Students Should Know | What All Students Should Be Able To Do | Sample Learning Activities |
|---|---|---|
| *By the end of grade 12, all students should know* | NOTE: Each item in this column is designed to address several elements of "what all students should be able to do."<br><br>*By the end of grade 12, all students should be able to* | NOTE: Each activity is designed to address several items from "what all students should know" and "what all students should be able to do." The activities also relate to strands other than patterns and relationships. |
| 1. Mathematical patterns and relationships may be represented in various forms. | a. compare and contrast the real number system and its various subsystems with regard to their structural characteristics (NCTM Standard 14; MO 1.6, 1.8) | • Use varied technologies or problem-solving techniques to explore the following situation: A heavy equipment operator makes an hourly wage of $15.00 per hour and a laborer makes an hourly wage of $10.00 per hour. The laborer starts preparing the job for work at 7:00 a.m. The heavy equipment operator starts work at 8:00 a.m. Each worker gets a 1-hour lunch break.<br><br>Explore ways to determine the time of day each worker will have earned the same amount of wages for the day. How much money will each worker have made at the end of 8 hours? How much money will each worker have made by 5:00 p.m.? (Workers receive time and one-half for working more than 8 hours in a day.) |
| 2. Mathematical symbols can be used to represent real-world situations. | b. represent and analyze relationships using verbal rules, tables and graphs as tools to interpret expressions, equations and inequalities (NCTM Standards 5 and 6; MO 1.6, 1.8, 2.1, 3.3) | |
| 3. Definitions of sequences and series. | | |
| 4. Trigonometric ratios of sine, cosine, and tangent. | c. translate among tabular, symbolic, and graphical representations of functions and model real-world phenomena with a variety of functions (NCTM Standard 6; MO 1.6, 1.8, 2.2, 3.6) | • Use varied technologies or problem-solving techniques to explore the following situation: A dog pen is being built beside the garage. The garage serves as one side of the fence. The amount of fence purchased is 20 feet. Explore ways to determine the maximum area for the fence. What length and width will give the maximum area? |
| 5. Subsets of the real number system. | | |
| | d. represent situations that involve variable quantities with expressions, equations and inequalities (NCTM Standard 5; MO 1.6, 1.8, 3.3) | • Design a diagram for a candy box that contains five different kinds of candy. Each box has at least three pieces of candy of the same kind. No two of the same kind of candy are next to each other on the same row, on the column or on the diagonal. Could the candy box be designed in more than one way? Justify your answer. |

# VIII. Patterns and Relationships

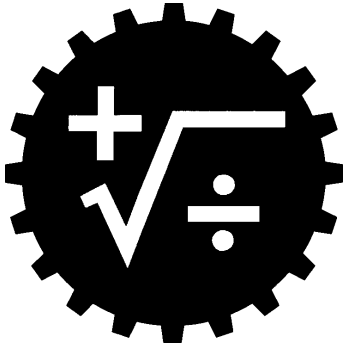
| What All Students Should Know | What All Students Should Be Able To Do | Sample Learning Activities |
|---|---|---|
| | e. solve equations and inequalities (NCTM Standard 5; MO 1.6, 1.8, 2.2, 3.3)<br><br>f. translate between synthetic and coordinate representation for geometric relationships (NCTM Standard 8; MO 1.6, 1.8, 2.2, 3.3)<br><br>g. investigate limiting processes by examining infinite sequences and series (NCTM Standard 13; MO 1.6, 1.8, 3.3)<br><br>h. apply trigonometry to problem situations involving triangles and explore real-world phenomena using the sine, cosine, and tangent functions (NCTM Standard 9; MO 1.6, 1.8, 2.2, 3.6)<br><br>i. analyze effects of parameter changes on the graphs of functions using a variety of technologies to gather data (NCTM Standard 6; MO 1.4, 1.6, 2.7, 3.3) | • A summer job consists of mowing yards for 16 weeks. The first week the pay is $0.25. The pay doubles each week. At what week will the pay be more than $100.00 for the first time? What would the pay be for mowing the lawn on the 16th week? Graph your results. Do you think that the employer can complete the payment for this plan all 16 weeks? Write a paragraph to justify your answer.<br><br>• Do the following problem:<br><br>The price of business cards are: STORE A - $30.00 for the first 200 cards and $0.05 for each additional card; STORE B - $0.10 for each card.<br><br>a. For what number of cards is the cost the same at both stores?<br><br>b. How many cards would you need to order so that STORE A has the best price?<br><br>c. Is there ever a situation where ordering less cards than those required for the breakeven point might be justified? Write a paragraph supporting your answer.<br><br>• Use graphing technology to explore families of algebraic expressions to determine if they are linear, non-linear, quadratic or cubic. |

| What All Students Should Know | What All Students Should Be Able To Do | Sample Learning Activities |
|---|---|---|
| | | • Play the Chaos Game. Draw an equilateral triangle. Label the vertices A, B, C, etc. Use a spinner or number cube to generate random numbers. On a number cube, vertex A could be represented by numbers 1 and 4. Vertex B could be represented by numbers 2 and 5. Vertex C could be represented by numbers 3 and 6. Start with any point in or around the triangle. Roll the cube or spin the spinner to determine a vertex. Measure one-half the distance between the point and the vertex. Make a dot. Generate a random number to indicate another vertex. Measure one-half the distance from the point to the vertex. Make a dot. Continue this procedure until a pattern starts to emerge. Do the activity on transparencies for the overhead projector. Put several groups of transparencies on top of the other to give a more complete pattern. Describe the pattern. (Adapted from *Making the Connections: Fractal Activities for the Secondary Classroom*, NCTM) |



MATHEMATICS 9–12

# IX. Mathematical Systems and Number Theory
# Show-Me Standards, Math 5



## K-12 Content Overview

Understanding the fundamental operations of addition, subtraction, multiplication and division is central to knowing mathematics. One essential component of what it means to understand an operation is recognizing conditions in real-world situations that indicate that the operation would be useful in those situations.

Children need extensive informal experience with problem situations and language prior to explicit instruction and symbolic work with the operations Thus, informal experience with all four operations should begin in kindergarten and continue through grade four. By emphasizing underlying concepts, using physical materials to model procedures, linking the manipulation of materials to the steps of the procedures, and developing think patterns, teachers can help children master basic facts and algorithms and understand their usefulness and relevance to daily situations.

In the middle school years, students come to recognize that numbers have multiple representations, so the development of concepts for fractions, ratios, decimals, and percents, and the idea of multiple representations of these numbers need special attention and emphasis. The ability to generate, read, use and appreciate multiple representations of the same quantity is a critical step in learning to understand and apply mathematics.

By the high school level, students should come to understand and appreciate mathematics as a coherent body of knowledge rather than a vast, perhaps bewildering, collection of isolated facts and rules. Understanding mathematical structure promotes students' efficiency in investigating the arithmetic of fractions, decimals, integers and rationals through the unity of common ideas. It also offers insights into how the whole number system is extended to the rational number system and beyond.

Relationships among operations are developed over many grades. At the K-4 level, multiplication can be viewed as repeated addition, and division can be considered as repeated subtraction. However, because the multiplication of fractions and decimals is not repeated addition and division is not always repeated subtraction, students should be exposed to other interpretations of multiplication and division as well. The development of personal meaning for numbers should be reinforced in the middle grades with an extension to other numbers and notations. Student experiences should include exploration of the properties of sets of numbers. High school students should extend their meaning of number to the real number system and a recognition that still other number systems exist. They should have the opportunity to develop intuitive proofs of properties of operations and sets of numbers, such as closure, commutativity and associativity.

| What All Students Should Know | What All Students Should Be Able To Do | Sample Learning Activities |
|---|---|---|
| *By the end of grade 4, all students should know*<br><br>1. Basic operations of addition, subtraction, multiplication and division are related to each other.<br><br>2. The concepts of factors and multiples in relation to multiplication and division. | NOTE: Each item in this column is designed to address several elements of "what all students should be able to do."<br><br>*By the end of grade 4, all students should be able to*<br><br>a. develop the "need" for whole numbers, integers and rational numbers, including fractions and decimals by looking for patterns and relationships to solve problems (NCTM Standard 6; MO 1.6, 3.2, 3,3)<br><br>b. develop and use number operations and order relations for decimals (money) (NCTM Standard 6; MO 1.6, 3.2, 3.3)<br><br>c. develop an understanding of how basic arithmetic operations are related to one another (NCTM Standard 6; MO 1.6)<br><br>d. develop and use number theory concepts, including factors and multiples in problem solving (NCTM Standard 6; MO 1.6, 3.5)<br><br>e. model, develop, and explain basic facts and algorithms with reasonable proficiency (NCTM Standard 8; MO 3.6) | NOTE: Each activity is designed to address several items from "what all students should know" and "what all students should be able to do." The activities also relate to strands other than mathematical systems and number theory.<br><br>• Use a set of manipulatives to explain the relationship between and among the operations of addition, subtraction, multiplication and division.<br><br>• Given a set of student-generated fraction or decimal cards, place the cards in order. Explain the process used.<br><br>• Represent all of the possible rectangles that can be made using 1-25 manipulatives (such as color tiles or unifix cubes). Explore and discuss the relationships of numbers that have multiple representations, a single representation, and can be represented by a square.<br><br>• Given a hundreds chart or 0-99 chart, search for patterns like doubles, odd numbers, even numbers, numbers with a 7 in them, numbers whose digits add 10, etc.<br><br>• Given a temperature of 20° at 8:00 p.m., develop a model to represent the temperature at 7:00 a.m. if the temperature drops an average of 4° per hour. |



MATHEMATICS K–4

## K. Mathematical Systems and Number Theory

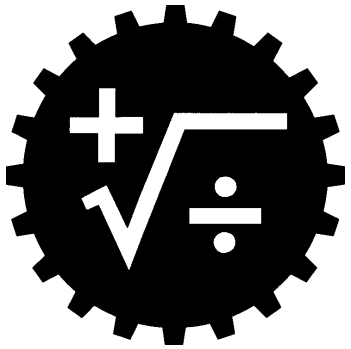
| What All Students Should Know | What All Students Should Be Able To Do | Sample Learning Activities |
|---|---|---|
| | | • Play "Oh, No, 49!" Deal 5 cards to each player. The player to the left of the dealer places a card in the center of the table, announcing the value of the card, and takes a card from the deck. The next player plays a card, adding the value of his/her card to the first card played. The player then also takes another card from the deck. Play continues, keeping a running total of cards played. Game ends when a player forces another player to exceed 49. Card values: A = 1; 2-9 = face value; 10= -10; J reverses operations (if you were adding, a jack causes the play to continue by subtracting); Q = 0; and K makes the total 49. |

| What All Students Should Know | What All Students Should Be Able To Do | Sample Learning Activities |
|---|---|---|
| *By the end of grade 8, all students should know*<br><br>1. Commutative, associative, and distributive properties.<br><br>2. Properties of zero and one.<br><br>3. Patterns may be used to describe relationships for multiples, factors, and exponents.<br><br>4. Order of operations. | NOTE: Each item in this column is designed to address several elements of "what all students should be able to do."<br><br>*By the end of grade 8, all students should be able to*<br><br>a. evaluate the need and applications of numbers not contained in the set of whole number (NCTM Standard 6; MO 1.1, 3.2, 3.3, 4.1)<br><br>b. develop an understanding of and explain order and relationship among integers, fractions, and decimals (NCTM Standard 6; MO 1.6, 2.2)<br><br>c. use real-world and mathematical problem situations to develop and apply number theory concepts (such as primes, factors, and multiples) (NCTM Standard 6; MO 1.6, 1.10, 3.2, 3.3)<br><br>d. realize the dynamic nature of mathematics and how different mathematical systems apply to current and developing real-world situations (NCTM Standard 6; MO 1.10, 2.5, 4.5, 4.8)<br><br>e. apply commutative, associative, and distributive relationships in computation and estimation situations. (NCTM Standard 6; MO 1.6; 3.2, 3.3)<br><br>f. recognize the connection of irrational numbers and the real world. (NCTM Standard 6; MO 1.6, 3.2, 3.3) | NOTE: Each activity is designed to address several items from "what all students should know" and "what all students should be able to do." The activities may also relate to strands other than mathematical systems and number theory.<br><br>• Investigate a problem situation requiring numbers not contained in the whole number set (such as figuring out the number of 70-seat buses necessary to transport 342 students or dividing a candy bar into 7 equal parts).<br><br>• Demonstrate a knowledge of order and relationship between various number sets through a problem situation. The activities could involve having each class member select a real number (including whole, natural, integer, rational or irrational numbers). Then make a number line so each person is in the proper order. Compare two similar numbers (such as 16.01 and 16.009876) by writing an explanation of each number's size and location on the number line.<br><br>• Describe, model, diagram, andor graph and the subsets of the real number set, such as whole, integer, and rational numbers. The description may include an investigation of real-life careers utilizing the various sets and an explanation of the ways the numbers are used in the real world.<br><br>• Use technology to begin an informal investigation of irrational numbers including, Pi, the square root of 2 or square root of 3 to discover where these types of numbers originated and how they are used in the real world.<br><br>• Investigate to determine which set of numbers is larger — rational or irrational. Then justify your conclusion. |

# IX. Mathematical Systems and Number Theory

| What All Students Should Know | What All Students Should Be Able To Do | Sample Learning Activities |
|---|---|---|
| *By the end of grade 12, all students should know*<br><br>1. Properties of real numbers.<br><br>2. Procedures for calculator and/or computer use.<br><br>3. Basic algebraic procedures.<br><br>4. Geometric relationships. | NOTE: Each item in this column is designed to address several elements of "what all students should be able to do."<br><br>*By the end of grade 12, all students should be able to*<br><br>a. compare and contrast the real number system and its various subsystems (NCTM Standard 14; MO 2.1, 4.1)<br><br>b. select and apply appropriate technology as a problem-solving tool to achieve understanding of the logic of algebraic and geometric procedures (NCTM Standard 14; MO 1.4, 3.6)<br><br>c. investigate and determine similarities and differences between mathematical systems (NCTM Standard 14; MO 2.1, 4.1, 4.6)<br><br>d. extend understanding and application of number theory concepts (NCTM Standard 6; MO 1.6, 3.2, 3.3) | NOTE: Each activity is designed to address several items from "what all students should know" and "what all students should be able to do." The activities may als relate to strands other than mathematical systems and number theory.<br><br>• Test for closure, commutative, associative, identity, inverse, and distributive properties in subsystems or the real numbers such as positive integers, integers, and rational numbers.<br><br>• Explore triangular numbers, primes, composites, factors, the Fibonacci sequence, Pascal's Triangle, etc., to enhance understanding of number relationships.<br><br>• Model algebraic procedures using various technologies such as tiles, balance, or graphing calculators.<br><br>• Use diagrams to demonstrate an understanding of the logic of conjunctions and disjunctions.<br><br>• Investigate sets of symmetries of geometric figures to compare their structures (for example, isosceles triangles and parallelograms).<br><br>• Investigate the structural commonalities of the binary system as it applies to multiplication, the logical operations of "and," and electrical current.<br><br>• Investigate geometry as a mathematical system by making and testing conjectures to arrive at generalizations.<br><br>• Develop and illustrate the use of your own mathematical system. |

# X. Discrete Mathematics
## Show-Me Standards, Math 6



### K-12 Content Overview

Many applications tied to the principles in business, to computer science, and to other real-world problems involve an area of mathematics called "discrete mathematics." The word *discrete* is defined as "separate or distinct." Discrete mathematics is the study of points, ideas and objects that are separate from each other or distinct. Discrete mathematics has many practical applications that are useful for solving some of the problems of society and that are meaningful to students. This area does not have extensive prerequisites, yet poses challenges to all students.

Students should learn to recognize examples of discrete mathematics in familiar settings; they should explore and solve a variety of problems for which discrete techniques have proved useful. Five major themes of discrete mathematics should be addressed at all grade levels: systematic listing, counting, and reasoning; discrete mathematical modeling using graphs and trees; iterative (repetitive) patterns and processes; organizing and processing information; and finding the best solution to problems using algorithms.

Discrete mathematics builds upon and extends the mathematics in the other nine strands of this document. Algorithmic thinking, graph theory, probability and counting techniques, mathematics of decision-making, matrices, and recursion are all included in this strand. Discrete mathematics is an exciting and appropriate vehicle for working toward and achieving the goal of educating informed citizens who are better able to function in an increasingly technological society, have better reasoning power and problem-solving skills, are aware of the importance of mathematics in our society, and are prepared for future careers that will require new and more sophisticated analytical and technical tools.

# X. Discrete Mathematics

| What All Students Should Know | What All Students Should Be Able To Do | Sample Learning Activities |
|---|---|---|
| *By the end of grade 4, all students should know*<br><br>1. Numbers in sequence to count objects.<br><br>2. Definition of "more" and "fewer."<br><br>3. Definition of "same" and "different."<br><br>4. Definition of "shortest" and "longest." | NOTE: Each item in this column is designed to address several elements of "what all students should be able to do."<br><br>*By the end of grade 4, all students should be able to*<br><br>a. determine what should be counted in a set of objects and actually count the objects (NCTM Standard 6; MO 1.8)<br><br>b. predict whether the set contain more or fewer of one subset than the other (NCTM Standard 6; MO 2.2)<br><br>c. illustrate or explain how the subsets of objects are the same or different (NCTM Standard 3; MO 1.8)<br><br>d. identify and discuss overlapping subsets of objects (Venn diagrams) (NCTM Standard 3; MO 2.2)<br><br>e. create algorithms based on constructing meaning from explorations (NCTM Standards 7 and 8; MO 1.6, 3.4, 3.6) | NOTE: Each activity is designed to address several items from "what all students should know" and "what all students should be able to do." The activities also relate to strands other than discrete mathematics.<br><br>• Use concrete materials (manipulatives) and/or diagrams to illustrate a variety of problem -solving situations with emphasis on thinking and common sense. Sequence then count a set of objects. Group the objects to count by 2s, 3s, 5s, and 10s. How does this relate to the concept of multiplication? How many different ways can three distinct object counters be arranged in a row? How do you know you have found all the ways? Explain or draw a diagram.<br><br>• Select a method of comparison and then sort concrete materials, pictures, or diagrams to match their rule. Make comparisons, looking for alike or different, more or less, smaller or larger, etc., then write about or tell about the method of comparison selected and how each part relates to the entire set of materials.<br><br>• Take concrete objects or pictures that have more than one attribute such as color, shape, or size. Then use a diagram to sort the objects using the rule that was selected. |

# X. Discrete Mathematics

| What All Students Should Know | What All Students Should Be Able To Do | Sample Learning Activities |
|---|---|---|
| | f. determine a path through a maze, whether a street network could be traveled going over each street one time, and the shortest distance traveling on a network of roads or streets (NCTM Standard 9; MO 2.2, 3.3, 3.4)<br><br>g. apply the concept of *fair division* to real-world situations (NCTM Standard 1; MO 2.2, 3.2, 3.3, 3.4, 3.7)<br><br>**fair division*: the division of an object (such as food among children or an estate among heirs) in a fair way for all people involved. | • Investigate algorithms in the study of mathematics to help organize and structure thinking. An algorithm is a sequence of instructions that if followed for an operation will always lead to a defined result. Use concrete materials to invent algorithms to solve addition, subtraction, multiplication, and division problems. Sequence a given set of pictures. Illustrate a sequence of instructions on how to get home from the school, or the order in which you do things when you are getting ready for school in the mornings.<br><br>• Draw a map of two routes to get from school to your home. Determine the shorter route, then talk about or write about why one way of going home is better than the other.<br><br>• Arrange attribute blocks or attrilinks into a train so each piece is next to another piece that is different from it in just one way.<br><br>• Given a collection of objects, sort the materials into two, three, or four sets (these sets could be placed within string loops). Record the attributes for each set on a sheet of paper. Have another group of students observe the sets and try to identify the attributes.<br><br>• Explore the idea of fair division by sharing a candy bar with a friend. Divide the candy bar into two equal pieces and have the friend pick the first piece. Is this fair or not? Devise a way to divide a cake or pizza to be shared by four friends. Is the method fair? Explain your answer. |

# X. Discrete Mathematics

| What All Students Should Know | What All Students Should Be Able To Do | Sample Learning Activities |
|---|---|---|
| *By the end of grade 8, all students should know*<br><br>1. Definition and example of patterns.<br>2. Definition and example of tree diagrams.<br>3. Definition and examples of Venn diagrams.<br>4. Definition and examples of networks. | NOTE: Each item in this column is designed to address several elements of "what all students should be able to do."<br><br>*By the end of grade 8, all students should be able to*<br><br>a. determine and continue a pattern using inductive reasoning (NCTM Standard 3; MO 1.6, 1.8, 3.5, 4.1)<br><br>b. look at if-then relationships to make logical deductions (NCTM Standard 3; MO 1.6, 3.5, 4.1)<br><br>c. investigate tree, Venn, or student-developed diagrams as an organizing tool for problem solving (NCTM Standard 3; MO 1.8, 2.1, 2.2)<br><br>d. explore transportation networks (NCTM Standards 2, 4, and 12; MO 2.2, 3.2, 3.3, 3.6) | NOTE: Each activity is designed to address several items from "what all students should know" and "what all students should be able to do." The activities may also relate to strands other than discrete mathematics.<br><br>• Extend a pattern made with blocks; make a table of the numerical values to represent the pattern. Determine the 10th term and the 100th term.<br><br>• Write statements such as, "If I go to the football game Friday evening, then I will not be home." Determine the relationship of the "if" part of the statement to the "then" part of the statement.<br><br>• Use diagrams as tools to find the solution of non-routine problems, such as the following: The 28 students in a homeroom class are having a pizza party. Fifteen students like pepperoni pizza and 17 students like hamburger pizza. All of the students like pepperoni, hamburger, or both kinds of pizza. How many students like both kinds of pizza?<br><br>• Use a local map that shows a portion of the area where you live. Design a route to deliver pizza to six of your friends. Design the route so that the driver will drive the least possible distance.<br><br>• Design a tournament bracket for a contest in the intramural program for your school by using a diagram. Do certain numbers work better for making a tournament bracket than others? If certain numbers work better, explain their characteristics. |

# X. Discrete Mathematics

| What All Students Should Know | What All Students Should Be Able To Do | Sample Learning Activities |
|---|---|---|
| *By the end of grade 12, all students should know*<br><br>1. Concepts of geometry.<br><br>2. Concepts of probability and statistics.<br><br>3. Characteristics of exponential growth.<br><br>4. Cause-and-effect.<br><br>5. Whole-to-part. | NOTE: Each item in this column is designed to address several elements of "what all students should be able to do."<br><br>*By the end of grade 12, all students should be able to*<br><br>a. explore and solve application problems involving graph theory (airline routes, circuits, paths, connecting roads, coloring a map, etc.) (NCTM Standard 12; MO 1.6, 1.8, 2.2, 3.2, 3.3, 3.6)<br><br>b. use tree, Venn, or student-developed diagrams as problem-solving tools (NCTM Standard 12; MO 3.2, 3.3, 3.6)<br><br>c. use concepts from logic and/or truth tables to recognize valid and invalid arguments (NCTM Standard 3; MO 2.2, 3.5)<br><br>d. explore applications from counting techniques such as Pascal's Triangle, permutations, combinations, and Fibonacci sequence (NCTM Standard 12; MO 1.6, 2.2, 3.6, 3.6) | NOTE: Each activity is designed to address several items from "what all students should know" and "what all students should be able to do." The activities may also relate to strands other than discrete mathematics.<br><br>• Design a shopping complex, a city park, or a trail containing intersecting paths or roads. The design must allow visitors to walk or drive through the entire area without going over the same trail or road twice. Visitors may go through an intersection more than one time with trails or roads previously traveled.<br><br>• Use diagrams as a way to visually account for all of the possibilities in a counting situation. Sample problem: A restaurant near your school offers dinners with four different meats and three different vegetables. The dinner contains one meat choice and two vegetable choices. How many different dinners could be ordered?<br><br>• Research how the apportionment in the House of Representatives is determined. Determine how it is possible for a state to gain population by the last census and lose numbers of representatives. Illustrate the mathematics involved in the apportionment process that would allow this to happen. Write a paragraph to explain the results of your research. |

# X. Discrete Mathematics

| What All Students Should Know | What All Students Should Be Able To Do | Sample Learning Activities |
|---|---|---|
| | e. investigate the concepts of *game theory** (NCTM Standard 1; MO 3.2, 3.7, 3.8)<br><br>**game theory*: selecting the best strategies in order to achieve the most favorable outcomes. Games are defined as have two or more players with conflicting interests.<br><br>f. explore concepts from election theory (NCTM Standard 1; MO 3.2, 3.7, 4.2, 4.3)<br><br>g. investigate different approaches to apportionment and fair division, then explore their applications (e.g., division of property in estates, apportionment in the House of Representatives) (NCTM Standard 1; MO 3.2, 3.3, 3.7, 4.3)<br><br>h. use the concept of recursion in mathematics to solve application problems (e.g., compound interest, depreciation, radium decay, maximum storage in the least amount of space, fractals) (NCTM Standard 12; MO 1.8, 2.2, 3.2, 3.7) | • Write a mathematical (recurrence) relationship that may be used to determine the amount of money in the savings account described below after n months. Determine the amount of money in the savings account at the end of 1 year.<br><br>James has $1,000.00 in a savings account when he finishes college and gets his first full-time job. He has included in his financial plan a monthly $50 deposit in a savings account. The annual interest rate is 4.8% compounded monthly. Complete the table below.<br><br>n (in months) $t_n$<br><br>0 $1,000<br>1 1.004(1000) + 50 = $1054<br>2<br>3<br>.<br>.<br>.<br>12<br>.<br>.<br>.<br>n |